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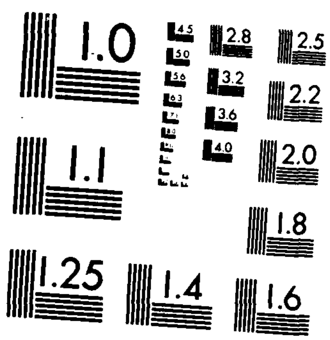


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During the period covered by the grant two research papers have been written and three others are in preparation. Titles include: "Identifiability Under Approximations for two-Point Boundary Value Problems", "Estimation of Elastic Parameters in Beams and Certail Plates: $H^1$ Regularizations", and "Shape Control for Static Beams and Plates.

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ANNUAL TECHNICAL REPORT

July 1, 1984 - July 1, 1985

AFOSR-TR- 86-0965

# Estimation and Control of Distributed Models for Certain Elastic Systems Arising in Large Space Structures

AFOSR Grant No. AFOSR-84-0271

Principal Investigator: Professor Luther W. White
Department of Mathematics
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1. Summary.

During the reporting period progress was made toward the goal of developing efficient and accurate estimation and control algorithms for elastic structures composed of beams and plates. Specifically, results were obtained that determine necessary and sufficient conditions for uniqueness in discrete versions of second order elliptic estimation problems. Moreover, theory and algorithms were developed for the estimation of elastic coefficients for static and dynamic models of beams and plates, the control of the shape of static beams and plates by means of actuators of small support, and the optimal placement of actuators to control the shape of beams and plates. These efforts have produced computer codes on which experimentation is currently being conducted to test methods and algorithms.

## 2. Research Objectives.

The research objective of this project is to study the estimation and control of elastic systems composed of beams and plates in order to develop efficient and accurate control and estimation algorithms. In estimation basic to this goal is to develop an understanding of properties of the parameter to state mappings, an approximation theory associated with particular models of interest, and the suitability of different minimization algorithms for efficient codes for various problems. In control of prime importance is to determine properties of optimal controls and feedback, best location based on design of the actuators and the geometry and elastic properties of the body, and suitable algorithms and codes. Toward these objectives the work during the past year has centered primarily on the estimation and control of static problems although codes for the estimation of elastic and damping parameters in dynamic plate models were also completed.

In considering the parameter to state mapping certainly an important property is injectivity. The injectivity and stability of this mapping is called identifiability. These properties are of course quite important for computational purposes and development of stable algorithms. Typically for distributed systems identifiability is rather difficult to establish. For applications the finite dimensional approximating models are closer to the computer models. An object is to study identifiability for these simpler models. Second order two point-boundary value problems were analyzed for various cases to obtain necessary and sufficient conditions for identifiability.

A second objective was to develop estimation algorithms for the estimation of elastic parameters in plates. Typically for a static problem a model of the form

(1) $$\Delta(a_0 \Delta u) - \nabla \cdot (a_1 \nabla u) + a_2 u = f \quad \text{in } \Omega$$

for $\Omega$ a bounded domain in $\mathbb{R}^2$ with appropriate boundary conditions was considered. The estimation problem may be stated as follows. Given an observation $z$ taken in an observation space $Z$ of the deformation find a parameter, for example $\hat{a}_0$, from within an admissible set $Q_{ad}$ in a parameter space $Q$ that minimizes a fit-to-data functional

(2) $$J(a_o) = \|Cu(a_0) - z\|_Z^2 + \varepsilon \|a_0\|_Q^2$$

over the set $Q_{ad}$. Here $C$ denotes an operator that takes the solutions $u$ in $X$ to the states $Cu(a_0)$ in $Z$. To solve this problem it is important to determine properties of the optimal estimators such as regularity and stability with respect to constraints and observations. The understanding of these properties aid in developing approximation theories for algorithms and codes.

A third objective over the last year was to develop control algorithms for the design of plates and beams of specified shape. Since the motivating problem was that of designing a mirror deformed to a specified shape and curvature, the control functional chosen for the first static examples was one related to the $H^2$-norm of the deformation. More precisely, given a bounded domain $\Omega \subset \mathbb{R}^2$ with the plate model

(3) $$\Delta(a \Delta u) = \sum_{i=1}^{\omega} \beta_i \Phi_i \quad \text{in } \Omega$$

with appropriate boundary conditions and $\Phi_i$ in $H^{-2}(\Omega)$, find $\hat{\beta} = (\hat{\beta}_1, \cdots, \hat{\beta}_\omega)$ in $\mathbb{R}^\omega$ that minimizes

$$J(\beta) = \int_\Omega (a\Delta(u(\beta)-z))^2 dx + \varepsilon \sum_{i=1}^{\omega} \beta_i^2 \tag{4}$$

for $\beta \in \mathbb{R}^\omega$ . The $\Phi_i$ determine the control mechanism and may represent control at a point, distributed along a curve, or over a subset of $\Omega$ of positive measure. Further, for the case of point control for design, a problem of interest is to determine optimal location of the actuators. These problems represent a first step and are to be followed by the consideration of the control of dynamic models.

3. Status of the Research.

In this section the status of the research outlined in Section 2 is indicated. In studying identifiability, research has concentrated on the study of identifiability for discrete models that arise as approximations of the distributed models. The discrete model is obtained once basis functions have been chosen to approximate the solution of the partial differential equation and the coefficient to be estimated. In work with Professor K. Kunisch a series of concrete cases was considered using a variety of basis functions. This research has dealt with second order elliptic two point boundary value problems, [1] in Section 4 and extended previous work for the parabolic equation entitled "Parameter Identifiability under Approximation" to appear in the _Quarterly of Applied Mathematics_. The results obtained in [1] give necessary and sufficient conditions in terms of rank conditions that may be calculated once basis functions for approximating subspaces have been specified. The techniques developed carry over directly to the fourth order elliptic problems.

The investigations into estimation for plates and beams have had several results. In this study to approximate the estimation problem one must translate the constraints that determine $Q_{ad}$ in a meaningful fashion to those for the finite dimensional problem. This may be accomplished in certain instances by considering regularized problems $\varepsilon > 0$ in (2). In addition, by absorbing constraints by introducing Lagrange multipliers one may deduce regularity properties of the optimal estimators. These properties are useful in translating the constraints and in applying the penalty method to approximating problems. These results are presented in [2,3,5,6] in Section 4. Further work has been carried out and codes have been written for dynamic problems which are currently being tested.

Concerning the control of plates and beams, the initial stages of research have been motivated by the problem of controlling the shape of a mirror. The model is that of a linear fourth order static plate equation (3). This problem may be viewed as a design problem. The work on this problem is outlined in Section 3 and appears in [4].

The controller $\Phi_i$ may be a function in $L^2(\Omega)$ with support contained in a ball centered at $x_i$ or may be a Dirac delta function $\delta_{x_i}$ with mass at $x_i$. Problem (3)-(4) has been coded in these cases with good results for simply supported boundary conditions. Cases with other boundary conditions are currently under study.

For the case $\Phi_i = \delta_{x_i}$ the optimal vector $\beta$ is in fact a function of $X$, the actuator locations. Hence, mappings $\Omega^\omega \to \mathbb{R}^\omega$ and $\Omega^\omega \to \mathbb{R}$ may be defined by $X \to \beta(X)$, where $\beta(X)$ represents the soltuion of (2), and $X \to j(X) = J(\beta(X))$, respectively. The question may be considered: Given a compact subset $F$ of $\Omega$ and a desired deformation $z$ is there an optimal set of actuator locations. This problem is analyzed in [4] by considering continuity and differentiability properties of the mappings defined above. Moreover, in [4] numerical results are given for the simple case of locating one actuator on a beam. Further, cases are under study with more actuators and for a plate. Finally, these results are being extended to include dynamic models of plates and beams.

4. Publications July 1, 1984 - July 1, 1985

1. Identifiability under Approximations for Two-Point Boundary Value Problems, with K. Kunisch, to appear SIAM J. Cont. and Opt.

2. Estimation of Ealstic Parameters in Beams and Certain Plates: $H^1$ Regularization, submitted to J. Opt. Thy. Appl.

3. Estimation of Elastic Coefficients in Beams: Penalization, submitted to J. Math. Anal. Appl.

4. Shape Control for Static Beams and Plates, in preparation.

5. Estimation of Damping and Elastic Parameters in Plates, in preparation.

6. Estimation of Elastic Parameters for a General Static Plates, in preparation.

5. List of Professional personnel.

(i) Paul Rye, Master of Science in Mechanical Engineering, June 1985.

(ii) Professor Karl Kunisch, Consultant.

6. Interactions.

(i) Talks:

Estimation of Parameters in Beams and Plates, ICASE at NASA Langley Research Center, June 1985.

Shape Control of Plates and Optimal Location of Actuators, Analysis Seminar, Unviersity of Oklahoma, Dept. of Mathematics, Spring 1985.

(ii) Consulted with professor Tinsley Odem, TICOM, University of Texas, Dept. of Engineering Mechanics.

7. New Discovies.

Research in the first year of this project has produced codes for the following:

(i) Estimation of elastic parameters for beams and plates for static models.

(ii) Estimation of elastic and damping parameters for dynamic models of plates.

(iii) Shape control for a plate.

(iv) Optimal location of actuators for the control of a beam.

(v) Codes for the estimation of boundary conditions and diffusion coefficients for steady state and transient diffusion equations.

New results include the following:

(i) Necessary and sufficient conditions for approximate identifiability.

(ii) Regularity properties for optimal estimators for beams and plates.

(iii) Differentiability and continuity properties of the optimal control with respect to actuator locations for static plate and beam shape control problems.

(iv) Approximation theory for regularized and constrained static beam and plate estimation problems.

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